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Abstract

A method for the development of mathematical models for dynamic systems with arbitrary nonlinearities from measured data is described. The method involves the use of neural networks as embedded processors in dynamic system simulation models. The technique is demonstrated through generation of models for anharmonic oscillators described by the Duffing Equation and the Van der Pol Equation from measured input/output data. It is shown that high quality models of these systems can be developed using this technique which are efficient in terms of model size. Using neural networks as embedded processors, accurate models of the Duffing Oscillator and the Van der Pol Oscillator were generated which contained eighteen parameters in each case. The architecture used requires that the neural networks perform only function fitting, a task to which they are well suited while integrators handle the modeling of energy storage by the system. This allows model parameter count to remain low, averting the undesirable high parameter counts sometimes associated with neural network based models. Model architecture, test problem specification, model optimization techniques used, quality of the models produced, practical applications and future work are discussed.

Introduction

The development of accurate models for systems exhibiting both nonlinear and dynamic behavior is a topic of considerable interest to professionals in a broad spectrum of fields, including signal processing, control, structural science, and ecology. The use of modern computing tools and algorithms such as neural networks and conjugate gradient learning have allowed the iterative solution of problems previously prohibitive in size and defiant of analytical solution. Linear dynamic systems, on the other hand, are well understood, with numerous tools available for their identification. This paper explores the use of neural networks as function fitting modules embedded in differential-equation-based models of nonlinear dynamic systems.

In the evaluation of a new modeling technique, simple systems offer an opportunity for such evaluation to be carried out with maximum comprehensibility of the results. The Duffing and Van der Pol Oscillators are singe-degree-of freedom (SDOF), second order, nonlinear processes which have been used as test problems for nonlinear system identification method evaluation previously (Masri, 1979), (Masri, 1993). For these reasons, the identification of these systems was chosen as the test problem set here.

In order to extend an SDOF linear model of a constant mass mechanical system to include nonlinearity, accomodation must be made for two possible sources of nonlinear behavior: nonlinear spring characteristics and nonlinear damper characteristics. A single neural network can be used to provide a function mapping capable of representing a spring-damper combination of arbitrary characteristic (Masters, 1993). This method has been demonstrated previously (Masri, 1993) in the successful development of a neural network based model for the Duffing Oscillator. In this work, it is demonstrated that the embedded network architecture gives much flexibility as the Duffing and Van der Pol Oscillators are here modeled using identical topology (only the parameters are changed); furthermore, accurate models can be constructed using very small neural networks, resulting in compact, high-quality system models. Usage of a few terms essential to the discussion will be clarified before proceeding further: A Linear System is one possessing the properties of additivity and homogeneity as commonly referred to in system theory. A Dynamic System is a history dependent system; for mechanical systems, this implies energy storage. Those processes which do not store energy will be referred to as static systems. A Time Invariant System is a process whose transfer function does not change with time. The work presented here will focus on the identification of systems representable as a mass-spring-damper combination, where the mass is constant, and the spring-damper combination may possess arbitrary characteristics. A schematic representation of such a system is shown in Fig. 1. The type of systems to be identified here are therefore nonlinear, dynamic, and time invariant.

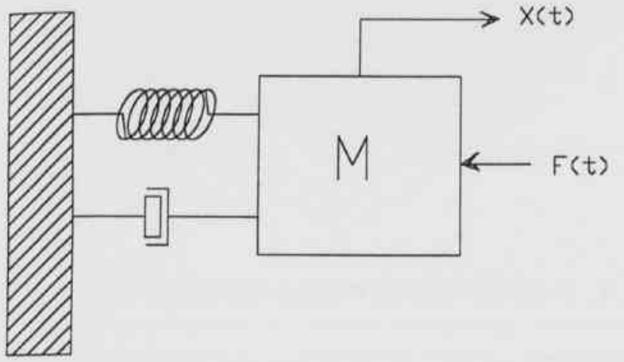

Fig. 1. Schematic representation of mass-spring-damper system.

Materials and Methods

Architecture of the Model.--The modeling method to be demonstrated is based on the well-known analog computer simulation architecture shown in Fig. 2. Here, integrators, adders and linear amplifiers are used to numerically solve the describing differential equation for a mass-spring-damper system:

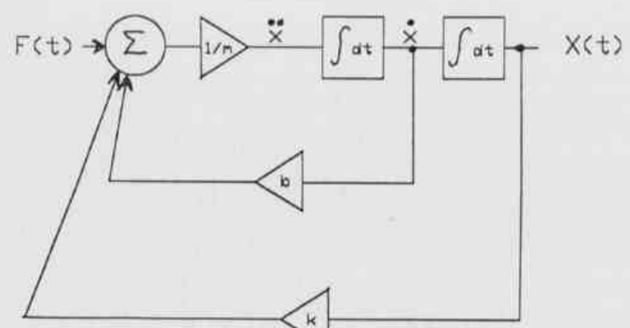

Fig. 2. Simulation architecture for a linear, second order, single degree of freedom system.

$F(t) = m\ddot{x} + b\dot{x} + kx$

where:

F(t) is the force input to the system (N),

x(t) is the position output (m),

m is the mass (kg),

b is the damping constant (N-s/m),

k is the spring constant (N/m).

In order to extend this model to accomodate systems with nonlinear spring and damper characteristics, the force terms $b\dot{x}$ and kx must be replaced by an appropriate nonlinear function of velocity and position. This mapping of position and velocity can be accomplished using a feedforward neural network as shown in Fig. 3.

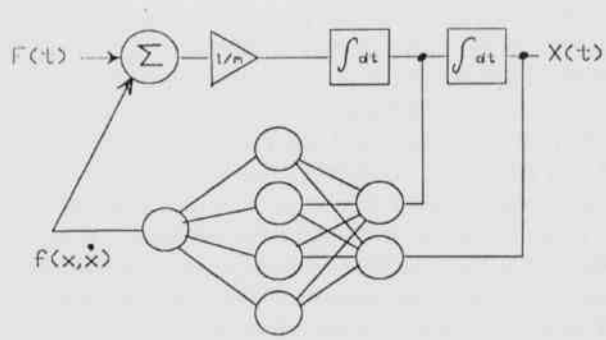

Fig. 3. Simulation model with embedded neural network for calculation of force function of nonlinear spring-damper combination.

Specification of the Test Problems.--In order to demonstrate the modeling technique, input and output data were generated through numerical solution of two well known nonlinear differential equations: the Duffing equation and the Van der Pol equation, listed below.

Duffing Equation

$F(t) = m\ddot{x} + b\dot{x} = k_1x + k_2x^3$

in the present study, this equation was modified to include a coupling term, resulting in a spring-damper characteristic that is nonlinear in both position and velocity, and thus a more challenging function fitting problem for the network.

$F(t) = m\ddot{x} + b\dot{x} + k_1x + k_2x^3 + a\dot{x}x^2$

where: m=500e-9, b=0.001, k_1=1.0, k_2=0.015, a=5e-6

These values were chosen to give an interesting amplitude and frequency response.

Van der Pol Equation

$F(t) = m\ddot{x} - a\dot{x}\,(1 - x^2) + x$

where: m=0.2, a=0.2
This is nonlinear in position and velocity. Parameter values were chosen to give an interesting amplitude and frequency response.

Model Optimization Techniques.--The method of training the networks to accurately map position and velocity to force for each oscillator to be identified is now described. If the force is due to a spring and damper of arbitrary characteristics is denoted as $f(x,\dot{x})$, the differential equation governing the motion of a constant mass system containing them becomes $F(t) = m\ddot{x} + f(x,\dot{x})$. Solving for this force gives $f(x,\dot{x}) = F(t) - m\ddot{x}$.

By recording many data points representing $F(t)$ and $x(t)$ and then numerically estimating the first and second time derivatives of $x(t)$ at each point, an array of data $x, \dot{x}, f(x,\dot{x})$ can be found for each data point. A subset (400 well-spaced points) of this list of inputs (position and velocity) and resulting output (force due to spring-damper combination) are used in the training of a feed-forward neural network that the network may learn an accurate mapping of the force function $f(x,\dot{x})$ of the system to be modeled. If the mass of the system is known or measurable, the model is completed. In the present study, the conjugate gradient algorithm (Masters, 1993) was used for training of the networks.

Probing Signals for Identification.--For each nonlinear oscillator to be identified, the training data was generated through excitation of the equation based simulation model with an amplitude modulated swept sine signal. This gave efficient coverage of the input space of the force function $f(x,\dot{x})$.

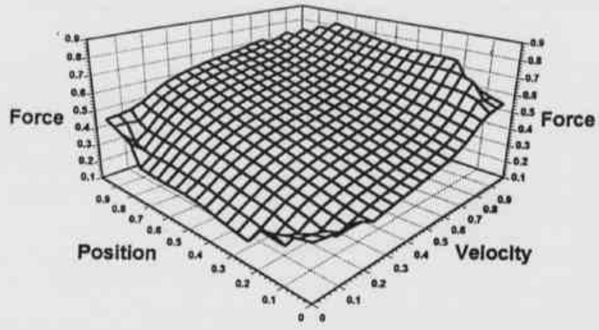

Fig. 4. Force function of Duffing Oscillator

Results

Duffing Oscillator.--The force function $f(x,\dot{x})$ measured from the Duffing oscillator simulation is graphically depicted in Fig. 4. The effect of the cubic term in the nonlinear spring is clearly visible as an increasing nonlinearity in x. Note that for a linear system, the force function will be planar. The time domain output of the neural-network-based model of the Duffing oscillator is compared with that of the equation-based reference system in Fig. 5.

Van der Pol Oscillator.--The force function $f(x,\dot{x})$ measured from the Van der Pol oscillator simulation is graphically depiced in Fig. 6. The effect of the coupling between the spring and damper is clearly visible. The time domain output of the neural-network-based model of the Van der Pol oscillator is compared with that of the equation-based reference system in Fig. 7.

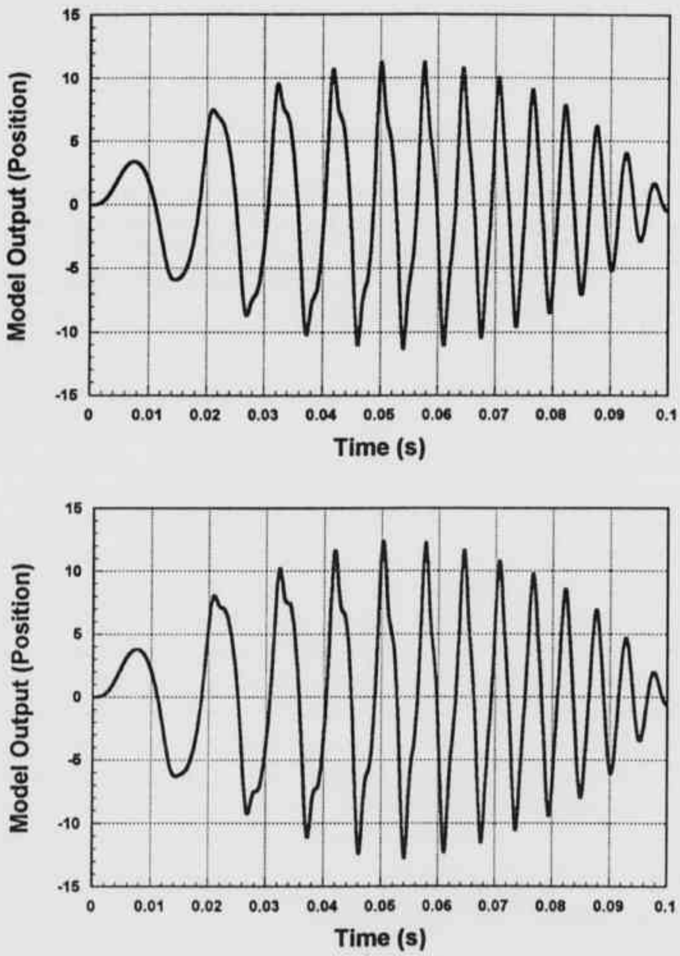

Fig. 5. Comparison of time domain response of reference system based on Duffing equation (top) with that of neural network based model (bottom).

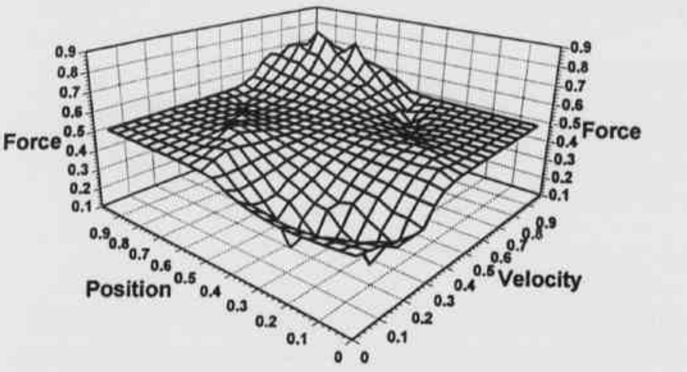

Fig. 6. Force function of Van der Pol Oscillator.

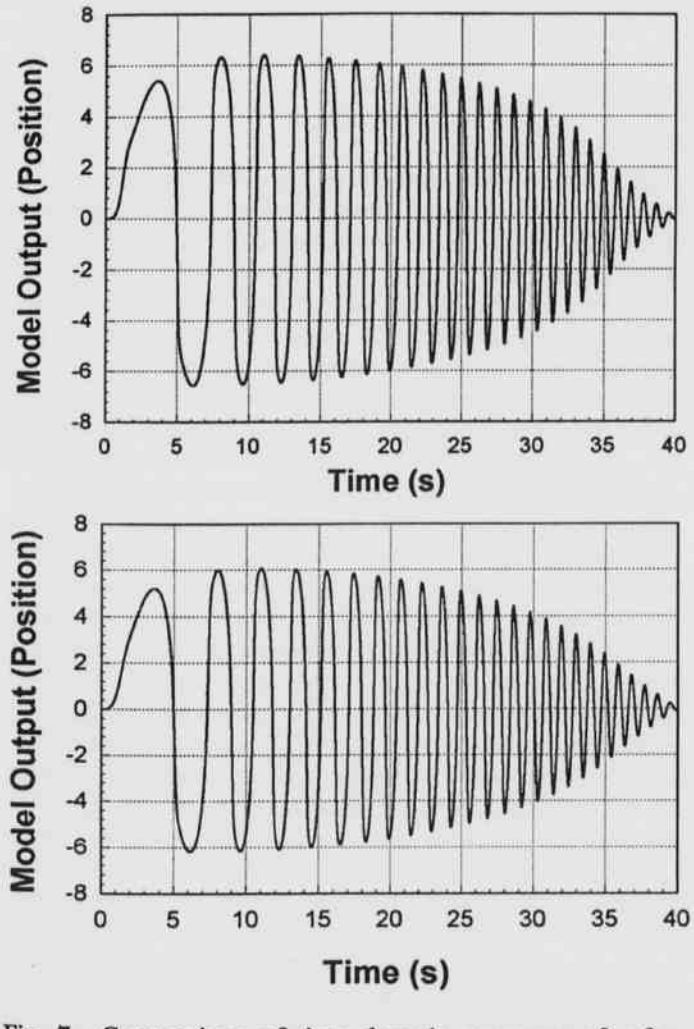

Fig. 7. Comparison of time domain response of reference system based on Van der Pol Equation (top) with that of neural network based model (bottom).

Discussion

Quality of the Models Produced.--As can be seen from Fig. 5, the time domain output of the neural-network-based Duffing oscillator model is very comparable to that of the equation-based reference model. In each case the input is an amplitude modulated swept sine signal. The output error can be seen to be small at all points. Similarly, good agreement is seen in Fig. 7 between the neural-network-based model of the Van der Pol oscillator and the equation-based reference model.

Complexity of the Models Produced.--In the Duffing oscillator model, the neural network employed contained two inputs, five hidden neurons, and one output neuron. This corresponds to seventeen network parameters (weights) in each model. This is slightly more complex than the orthogonal-polynomial-expansion-based model reported in (Masri, 1979), which used eight, but less complex than the normal network based model reported in (Masri, 1993) which contained 216. In the Van der Pol oscillator model, the neural network again employed seventeen network parameters (weights). This is less complex than the orthogonal-polynomial-expansion-based model reported in (Masri, 1979), which used 64. No precedent for comparison is found in the literature for a neural network based Van der Pol oscillator model. The low complexity of the model is most likely attributable to the assignment of tasks within it; feedforward neural networks perform static nonlinear mappings well, and an integrator is perhaps the most basic example of an energy storing device. Since these are the tasks here assigned, each component is utilized efficiently.

Complexity of Implementation of the Modeling Method.--The method was found to be easy to implement by the author, largely due to the fact that the architecture did not need to be changed from one problem to the next (only the network weights were changed), and because the training of these networks requires no special mathematical aptitude.

Applications of the Modeling Method.--Perhaps the most interesting application of nonlinear dynamic system modeling from measured data is the identification of a poorly understood process for the purposes of behavioral prediction or control. Possession of an accurate system model allows estimation of the response of the reference system to a hypothetical input, as well as offering benefit in the design of a controller for the reference system.

Future Work.--The next step in this work will be the development of a method for on-line optimization (as opposed to the training of the force function mapping network off-line as in the present work). Greater accuracy, as well as accommodation of time variance in the reference system could result. With on-line training, new training data can be incorporated into the training data set as the reference system is observed, allowing the model to "track" gradual changes in the behavior of the reference system. The application of the modeling method to the development of controllers for nonlinear dynamic systems is an area of possible future work also.

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